Algebraic Model for Thermal Dispersion Heat Flux within Porous Media

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Introduction

Thermal dispersion refers to the effect of the pore-level velocity nonuniformity on the temperature field within fluid-saturated porous media. The thermal dispersion dominates over the molecular diffusion as the flow velocity in porous media becomes sufficiently high. Thus, it is essential to account for its effect on heat-transfer characteristics in fluid-saturated porous media.

According to Wakao and Kaguei,[1] Yagi et al.[2] were the first to measure the effective longitudinal thermal conductivities of packed bed, taking full account of the effect of thermal dispersion, and eventually found the longitudinal component of the dispersion coefficient much greater than its transverse component. Since the famous analytical treatment in a tube by Taylor,[3] a number of theoretical and experimental efforts (for example, Aris,[4] Koch and Brady,[5] Han et al.,[6] and Vortmeyer[7]) were made to establish useful correlations for estimating the effective thermal conductivities due to thermal dispersion (See Kaviany[8]). Furthermore, a series of numerical experiments were conducted by Kuwahara et al.[9] and Kuwahara and Nakayama,[10] assuming a macroscopically uniform flow through a lattice of rods, so as to elucidate the effects of microscopic velocity and temperature fields on the thermal dispersion.

A number of theoretical models based on the volume averaging theory (for example, Cheng[11] and Nakayama[12]) have been proposed for both local-equilibrium and local-nonequilibrium situations in energy transport in porous media (for example, Kaviany,[8] Quintard and Whitaker,[13] Golfier et al.[14] and Nakayama et al.[15]). In most of the correlations available for the thermal dispersion flux, a gradient-type diffusion hypothesis has been adopted. In this study, we shall first derive a transport equation for the dispersion heat flux transport equation applying the volume averaging theory to the continuity equation, Navier-Stokes equation and energy equation, and then reduce it to obtain algebraic expressions for individual thermal dispersion coefficients. To authors' knowledge, this is the first time that the validity of such intuitive gradient-type hypotheses has been thoroughly examined in terms of the dispersion heat flux transport equation.

Volume Averaging Theory

In the volume averaging theory, we shall consider a control volume V in a fluid-saturated porous medium, as shown in Figure 1. The characteristic length of the control volume $V^{1/3}$ is much smaller than the macroscopic characteristic length $V_c^{1/3}$, but, at the same time, much greater than the microscopic (porous structure) characteristic length, for the spatial averaging (smoothing) process to be meaningful. Under this condition, the volume average of some variable ϕ is defined as

$$\langle \phi \rangle \equiv \frac{1}{V} \int_{V_f} \phi dV \tag{1}$$

where V_f is the volume space which the fluid occupies. Another averaging, namely, the intrinsic average of ϕ is defined as

$$\langle \phi \rangle^f \equiv \frac{1}{V_f} \int_{V_f} \phi dV \tag{2}$$

The following Dupuit-Forchheimer relation holds between the two averages

$$\langle \phi \rangle = \varepsilon \langle \phi \rangle^f \tag{3}$$

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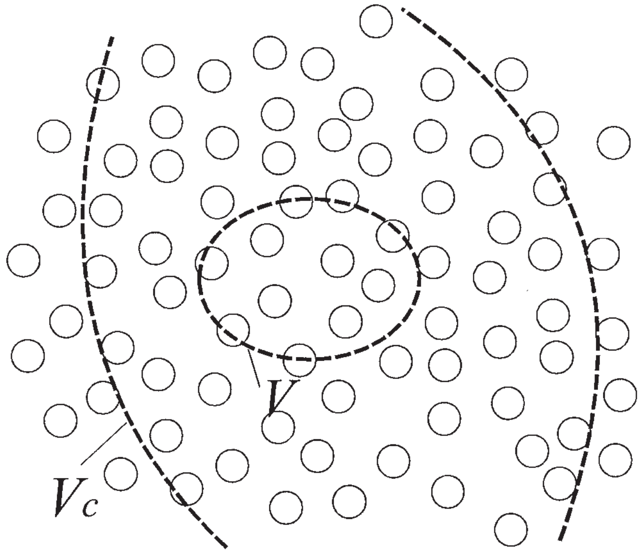

Figure 1. Microscopic view of porous structure.

where $\varepsilon \equiv V_f/V$ is the porosity. Any variable can be decomposed as the summation of the intrinsic average and the spatial deviation from it

$$\phi = \langle \phi \rangle^f + \tilde{\phi} \tag{4}$$

and it can easily be shown that

$$\langle \phi_1 \phi_2 \rangle^f = \langle \phi_1 \rangle^f \langle \phi_2 \rangle^f + \langle \tilde{\phi}_1 \tilde{\phi}_2 \rangle^f \tag{5}$$

Furthermore, the following theorems are known for integrations of derivatives or

$$\langle \nabla \phi \rangle = \nabla \langle \phi \rangle + \frac{1}{V} \int_{A_{\mathrm{int}}} \phi d\mathbf{A} \quad \text{or} \quad \langle \nabla \phi \rangle^f = \frac{1}{\varepsilon} \nabla \varepsilon \langle \phi \rangle^f + \frac{1}{V_f} \int_{A_{\mathrm{int}}} \phi d\mathbf{A} \tag{6a,b}$$

and

$$\left\langle \frac{\partial \phi}{\partial t} \right\rangle = \frac{\partial \langle \phi \rangle}{\partial t} \tag{7}$$

where A_{int} is the local interface between the fluid and solid, while $d\mathbf{A}$ is its vector element pointing outward from the fluid side to solid side. The foregoing relations will be exploited to derive the dispersion heat flux transport equation from Navier-Stokes equation. Note that the volume averaging procedure is somewhat more complex than the time averaging procedure used in the study of turbulence, since it involves with surface integrals, as clearly seen from Eqs. 6a and 6b.

Volume Averaged Governing Equations

In order to obtain the macroscopic governing equations, we shall integrate spatially the microscopic governing equations, namely, the continuity equation, Navier-Stokes equation and energy equations for two phases

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{8}$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} u_j u_i = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \nu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{9}$$

$$\rho_f c_{p_f} \left(\frac{\partial T}{\partial t} + \frac{\partial}{\partial x_j} u_j T \right) = \frac{\partial}{\partial x_j} \left(k_f \frac{\partial T}{\partial x_j} \right) \tag{10}$$

$$\rho_s c_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_j} \left(k_s \frac{\partial T}{\partial x_j} \right) \tag{11}$$

where the subscripts f and s stand for the fluid and solid, respectively. It is assumed that the fluid is incompressible and all properties are constant. These governing Eqs. 8 to 11 are integrated spatially over an elementary control volume V to yield the following macroscopic governing equations

$$\frac{\partial \langle u_j \rangle^f}{\partial x_j} = 0 \tag{12}$$

$$\frac{\partial \langle u_i \rangle^f}{\partial t} + \frac{\partial}{\partial x_j} \langle u_j \rangle^f \langle u_i \rangle^f = -\frac{1}{\rho_f} \frac{\partial \langle p \rangle^f}{\partial x_i} + \frac{\partial}{\partial x_j} \nu_f \left(\frac{\partial \langle u_i \rangle^f}{\partial x_j} + \frac{\partial \langle u_j \rangle^f}{\partial x_i} \right) + \frac{1}{V_f} \int_{A_{\mathrm{int}}} \left(-\frac{p}{\rho} + \nu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right) n_j dA - \frac{\partial}{\partial x_j} \langle \tilde{u}_j \tilde{u}_i \rangle^f \tag{13}$$

$$(\varepsilon \rho_f c_{p_f} + (1-\varepsilon) \rho_s c_s) \frac{\partial \langle T \rangle^f}{\partial t} + \varepsilon \rho_f c_{p_f} \frac{\partial}{\partial x_j} \langle u_j \rangle^f \langle T \rangle^f = \frac{\partial}{\partial x_j} \times \left((\varepsilon k_f + (1-\varepsilon) k_s) \frac{\partial \langle T \rangle^f}{\partial x_j} + \frac{k_f - k_s}{V} \int_{A_{\mathrm{int}}} T n_j dA - \varepsilon \rho_f c_{p_f} \langle \tilde{u}_j \tilde{T} \rangle^f \right) \tag{14}$$

where $\langle T \rangle^s$ is the intrinsic average of the solid temperature, and n_j is the unit vector pointing outward from the fluid side to solid side. The porosity ε is assumed to be constant. Moreover, the no-slip conditions are used over the interface of the rigid solid structure. The energy Eqs. 10 and 11 are integrated, and then combined together to form Eq. 14, assuming the condition of local thermal equilibrium $\langle T \rangle^{\mathrm{f}} = \langle T \rangle^{\mathrm{s}}$, which holds in most cases of flow in fluid-saturated porous media. Note that the dispersion heat flux $-\rho_f c_{pf} \langle \tilde{u}_j \tilde{T} \rangle = -\varepsilon \rho_f c_{pf} \langle \tilde{u}_j \tilde{T} \rangle^f$ appears in the volume averaged energy Eq. 14.

In order to close the foregoing macroscopic Eqs. 12 to 14, the terms associated with the surface integral are modeled according to Vafai and Tien[16] and Nakayama et al.[12] as

$$\frac{1}{V_f}\int_{A_{\rm int}}\left(-\frac{p}{\rho_f}+\nu_f\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right)n_j dA-\frac{\partial}{\partial x_j}\langle\tilde{u}_j\tilde{u}_i\rangle^f=-\frac{\nu_f}{K}\varepsilon\langle u_i\rangle^f - b\varepsilon^2(\langle u_k\rangle^f\langle u_k\rangle^f)^{1/2}\langle u_i\rangle^f \quad (15)$$

which is the well-known Forchheimer-extended Darcy law, where K and b are the permeability and Forchheimer constant, respectively. The term $((k_f - k_s)/V)\int_{A_{\rm int}} Tn_j dA$ describes the tortuosity heat fluxes, which are usually small and may well be neglected, as convection dominates over conduction. Hence, the set of macroscopic equations may be written as

$$\frac{\partial\langle u_i\rangle^f}{\partial t}+\frac{\partial}{\partial x_j}\langle u_j\rangle^f\langle u_i\rangle^f=-\frac{1}{\rho}\frac{\partial\langle p\rangle^f}{\partial x_i}+\frac{\partial}{\partial x_j}\nu\left(\frac{\partial\langle u_i\rangle^f}{\partial x_j}+\frac{\partial\langle u_j\rangle^f}{\partial x_i}\right) - \frac{\nu}{K}\varepsilon\langle u_i\rangle^f - b\varepsilon^2(\langle u_k\rangle^f\langle u_k\rangle^f)^{1/2}\langle u_i\rangle^f \quad (16)$$

$$(\varepsilon\rho_f c_{p_f}+(1-\varepsilon)\rho_s c_s)\frac{\partial\langle T\rangle^f}{\partial t}+\varepsilon\rho_f c_{p_f}\frac{\partial}{\partial x_j}\langle u_j\rangle^f\langle T\rangle^f=\frac{\partial}{\partial x_j} \times\left((\varepsilon k_f+(1-\varepsilon)k_s)\frac{\partial\langle T\rangle^f}{\partial x_j}-\varepsilon\rho_f c_{p_f}\langle\tilde{u}_j\tilde{T}\rangle^f\right) \quad (17)$$

In order to close the foregoing set of macroscopic equations, we need only determine the dispersion heat flux. In what follows, we shall derive the transport equation for the vector from which we can determine the thermal dispersion coefficients.

Dispersion Heat Flux Transport Equation

In order to derive the dispersion heat flux transport equation, we first subtract the macroscopic Eqs. 12, 16 and 17 from the microscopic Eqs. 8, 9 and 10, respectively, and obtain the corresponding transport equations for the spatial deviations as follows

$$\frac{\partial\tilde{u}_j}{\partial x_j}=0 \quad (18)$$

$$\frac{D\tilde{u}_i}{Dt}+\frac{\partial}{\partial x_j}(\tilde{u}_j\langle u_i\rangle^f+\tilde{u}_i\tilde{u}_j)=-\frac{1}{\rho_f}\frac{\partial\tilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\nu_f\left(\frac{\partial\tilde{u}_i}{\partial x_j}+\frac{\partial\tilde{u}_j}{\partial x_i}\right) + \frac{\nu_f}{K}\varepsilon\langle u_i\rangle^f + b\varepsilon^2(\langle u_k\rangle^f\langle u_k\rangle^f)^{1/2}\langle u_i\rangle^f \quad (19)$$

$$\frac{D\tilde{T}}{Dt}+\frac{\partial}{\partial x_j}(\tilde{u}_j\langle T\rangle^f+\tilde{u}_j\tilde{T}-\langle\tilde{u}_j\tilde{T}\rangle^f)=\frac{\partial}{\partial x_j}\left(\alpha_f\frac{\partial\tilde{T}}{\partial x_j}\right) \quad (20)$$

where

$$\frac{D\phi}{Dt}\equiv\frac{\partial\phi}{\partial t}+\langle u_j\rangle^f\frac{\partial\phi}{\partial x_j} \quad (21)$$

is a shorthand notation for the substantial derivative based on the intrinsic velocity. Upon noting the obvious relation

$$\frac{D\tilde{u}_i\tilde{T}}{Dt}=\tilde{T}\frac{D\tilde{u}_i}{Dt}+\tilde{u}_i\frac{D\tilde{T}}{Dt} \quad (22)$$

we formulate the terms on the righthand side of the foregoing equation using the transport Eqs. 19 and 20 as

$$\frac{\partial\tilde{u}_i\tilde{T}}{\partial t}+\frac{\partial}{\partial x_j}\left(\langle u_j\rangle^f\tilde{u}_i\tilde{T}+\tilde{T}\tilde{u}_i\tilde{u}_j-\alpha_f\tilde{u}_i\frac{\partial\tilde{T}}{\partial x_j}\right)=-\tilde{T}\tilde{u}_j\frac{\partial\langle u_i\rangle^f}{\partial x_j} - \tilde{u}_i\tilde{u}_j\frac{\partial\langle T\rangle^f}{\partial x_j}+\tilde{T}\left(-\frac{1}{\rho_f}\frac{\partial\tilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\nu_f\left(\frac{\partial\tilde{u}_i}{\partial x_j}+\frac{\partial\tilde{u}_j}{\partial x_i}\right)\right)-\alpha_f\frac{\partial\tilde{u}_i}{\partial x_j}\frac{\partial\tilde{T}}{\partial x_j} + \tilde{u}_i\frac{\partial}{\partial x_j}\langle\tilde{u}_j\tilde{T}\rangle+\tilde{T}\left(\frac{\nu_f}{K}\varepsilon\langle u_i\rangle^f+b\varepsilon^2(\langle u_k\rangle^f\langle u_k\rangle^f)^{1/2}\langle u_i\rangle^f\right) \quad (23)$$

where the deviating continuity Eq. 18 has been exploited. Then, carrying out the volume averaging treatment under the no-slip condition over the interface, we derive the following transport equation for the dispersion heat flux after some manipulations

$$\begin{aligned}
&\frac{D\langle\tilde{u}_i\tilde{T}\rangle^f}{Dt} && \text{:Convection}\\
&+\frac{\partial}{\partial x_j}\left(\langle\tilde{T}\tilde{u}_i\tilde{u}_j\rangle^f-\alpha_f\left\langle\tilde{u}_i\frac{\partial\tilde{T}}{\partial x_j}\right\rangle^f\right) && \text{:Diffusion}\\
&=-\left(\langle\tilde{T}\tilde{u}_j\rangle^f\frac{\partial\langle u_i\rangle^f}{\partial x_j}+\langle\tilde{u}_i\tilde{u}_j\rangle^f\frac{\partial\langle T\rangle^f}{\partial x_j}\right) && \text{:Production}\\
&-\alpha_f\left\langle\frac{\partial\tilde{u}_i}{\partial x_j}\frac{\partial\tilde{T}}{\partial x_j}\right\rangle^f && \text{:Dissipation}\\
&+\left\langle\tilde{T}\left(-\frac{1}{\rho_f}\frac{\partial\tilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\nu_f\left(\frac{\partial\tilde{u}_i}{\partial x_j}+\frac{\partial\tilde{u}_j}{\partial x_i}\right)\right)\right\rangle^f && \text{:Redistribution}
\end{aligned} \quad (24)$$

Algebraic Dispersion Heat Flux Model

The convection and diffusion terms on the lefthand side of Eq. 24 represent the spatial transport of the dispersion heat flux. As the divergence theorem indicates, these terms can influence the overall aspect of the thermal dispersion only through the events occurring on the boundaries. It is the first term on the righthand side that is responsible for generating the dispersion heat flux by the gradients of the volume averaged temperature and velocity, and thus, the term may be called the *production* term. The analogy between the dispersion heat flux and the turbulent heat flux indicates that the second on the righthand side corresponds with the *dissipation* of the dispersion heat flux, and that it is small enough to be neglected. Hence, it is the last term on the righthand side, namely, the *redistribution* term that virtually balances with the foregoing production term. Thus, for the first approximation, we neglect the spatial transport terms and dissipation term to obtain the following algebraic equation

$$-\left(\langle\tilde{T}\tilde{u}_j\rangle^f\frac{\partial\langle u_i\rangle^f}{\partial x_j}+\langle\tilde{u}_i\tilde{u}_j\rangle^f\frac{\partial\langle T\rangle^f}{\partial x_j}\right)+\left\langle\tilde{T}\left(-\frac{1}{\rho_f}\frac{\partial\tilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\nu_f\times\left(\frac{\partial\tilde{u}_i}{\partial x_j}+\frac{\partial\tilde{u}_j}{\partial x_i}\right)\right)\right\rangle^f=0 \quad (25)$$

The redistribution term needs some modeling. In order to estimate the second term in the foregoing equation, we integrate the deviating momentum Eq. 19 over the volume, applying the no-slip condition, and find

$$\left\langle-\frac{1}{\rho_f}\frac{\partial\tilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\nu_f\left(\frac{\partial\tilde{u}_i}{\partial x_j}+\frac{\partial\tilde{u}_j}{\partial x_i}\right)\right\rangle^f=-\frac{\nu_f}{K}\varepsilon\langle u_i\rangle^f-b\varepsilon^2(\langle u_k\rangle^f\langle u_k\rangle^f)^{1/2}\langle u_i\rangle^f+\left\langle\tilde{u}_j\frac{\partial\tilde{u}_i}{\partial x_j}\right\rangle^f \quad (26)$$

Therefore, we may estimate the term as

$$\left\langle\tilde{T}\left(-\frac{1}{\rho_f}\frac{\partial\tilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\nu_f\left(\frac{\partial\tilde{u}_i}{\partial x_j}+\frac{\partial\tilde{u}_j}{\partial x_i}\right)\right)\right\rangle^f\sim-\left\langle\tilde{T}\left\{\varepsilon\left(\frac{\nu_f}{K}+b\varepsilon(\langle u_k\rangle^f\langle u_k\rangle^f)^{1/2}\right)u_i-\tilde{u}_j\frac{\partial\tilde{u}_i}{\partial x_j}\right\}\right\rangle^f \quad (27)$$

Thus, we propose the following correlation for the redistribution term

$$\left\langle\tilde{T}\left(-\frac{1}{\rho_f}\frac{\partial\tilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\nu_f\left(\frac{\partial\tilde{u}_i}{\partial x_j}+\frac{\partial\tilde{u}_j}{\partial x_i}\right)\right)\right\rangle^f=-C_1\varepsilon\langle\tilde{u}_i\tilde{T}\rangle^f\times\left(\frac{\nu_f}{K}+b\varepsilon(\langle u_k\rangle^f\langle u_k\rangle^f)^{1/2}\right)+C_2\langle\tilde{u}_j\tilde{T}\rangle^f\frac{\partial\langle u_i\rangle^f}{\partial x_j} \quad (28)$$

where C_1 and C_2 are empirical constants. According to the theory of turbulence, the first and second terms correspond with the "slow" and "rapid" terms, respectively. Substituting Eq. 28 into Eq. 25, we obtain the algebraic dispersion heat flux model as

$$-(1-C_2)\langle\tilde{u}_j\tilde{T}\rangle^f\frac{\partial\langle u_i\rangle^f}{\partial x_j}-\langle\tilde{u}_i\tilde{u}_j\rangle^f\frac{\partial\langle T\rangle^f}{\partial x_j}-C_1\gamma\langle\tilde{u}_i\tilde{T}\rangle=0 \quad (29)$$

where

$$\gamma\equiv\varepsilon\left(\frac{\nu_f}{K}+b\varepsilon(\langle u_k\rangle^f\langle u_k\rangle^f)^{1/2}\right) \quad (30)$$

Equation 29 may be modified as

$$\left(C_1\gamma\delta_{ij}+(1-C_2)\frac{\partial\langle u_i\rangle^f}{\partial x_j}\right)\langle\tilde{u}_j\tilde{T}\rangle^f=-\langle\tilde{u}_i\tilde{u}_j\rangle^f\frac{\partial\langle T\rangle^f}{\partial x_j} \quad (31)$$

The foregoing expression turns out to be similar to a class of proposals found in the study of turbulent heat flux modeling, such as in Suga and Abe,[17] Launder[18] and Craft and Launder.[19] The inversion of this matrix equation gives a set of explicit expressions for three dispersion heat flux components. However, a further simplification is possible. We note that the first diagonal term $C_1\gamma\delta_{ij}$ is likely to predominate over the second term. $(1-C_2)(\partial\langle u_i\rangle^f/\partial x_j)$, since $\partial\langle u_i\rangle^f/\partial x_j$ is not significant in porous media, such as in the case of macroscopically uniform flow in a packed bed. Thus, dropping. $(1-C_2)(\partial\langle u_i\rangle^f/\partial x_j)$, we have a simple expression as follows

$$\langle\tilde{u}_i\tilde{T}\rangle^f=-\frac{\langle\tilde{u}_i\tilde{u}_j\rangle^f}{C_1\gamma}\frac{\partial\langle T\rangle^f}{\partial x_j} \quad (32)$$

Discussion

For a macroscopically unidirectional flow with the Darcian velocity u_D, namely, $\langle u_i\rangle^f=(u_D/\varepsilon,\ 0,\ 0)$, the foregoing expressions reduce to

$$-\rho_f c_{p_f}\langle\tilde{u}\tilde{T}\rangle\simeq\rho_f c_{p_f}\varepsilon\frac{\langle\tilde{u}^2\rangle^f}{C_1\gamma}\frac{\partial\langle T\rangle^f}{\partial x} \quad (33a)$$

$$-\rho_f c_{p_f}\langle\tilde{v}\tilde{T}\rangle\simeq\rho_f c_{p_f}\varepsilon\frac{\langle\tilde{v}^2\rangle^f}{C_1\gamma}\frac{\partial\langle T\rangle^f}{\partial y} \quad (33b)$$

Thus, the thermal dispersion coefficients are given by

$$\frac{(k_{dis})_{xx}}{k_f}=\frac{\langle\tilde{u}^2\rangle^f}{\alpha_f C_1\left(\frac{\nu_f}{K}+bu_D\right)}\simeq C_{xx}\frac{(1-\varepsilon)^{3/2}u_D^2}{\varepsilon^2\alpha_f\left(\frac{\nu_f}{K}+bu_D\right)} \quad (34a)$$

$$\frac{(k_{dis})_{yy}}{k_f}=\frac{\langle\tilde{v}^2\rangle^f}{\alpha_f C_1\left(\frac{\nu_f}{K}+bu_D\right)}\simeq C_{yy}\frac{(1-\varepsilon)^{3/2}u_D^2}{\varepsilon^2\alpha_f\left(\frac{\nu_f}{K}+bu_D\right)} \quad (34b)$$

where $\sqrt{\langle\tilde{u}^2\rangle^f}$, $\sqrt{\langle\tilde{v}^2\rangle^f}\propto u_D(1-\varepsilon)^{3/4}/\varepsilon$ is assumed according to the numerical experiments recently carried out by our group. In order to fit the proposed correlation with available data, we assume $C_{xx}\simeq 4.5$, and apply Ergun's empirical expressions[20] for K and b valid in a packed bed of the particle dia. d as

$$K=\frac{\varepsilon^3 d^2}{150(1-\varepsilon)^2}\quad\text{and}\quad b=1.75\frac{1-\varepsilon}{\varepsilon^3 d} \quad (35a,b)$$

Thus, we obtain

$$\frac{(k_{dis})_{xx}}{k_f}\simeq\frac{4.5\left(\frac{\varepsilon}{150(1-\varepsilon)^{1/2}}\right)Pe_d^2}{Pr+\left(\frac{1.75}{150(1-\varepsilon)}\right)Pe_d} \quad (36)$$

where $Pe_d=u_D d/\alpha_f$ is the Peclet number based on the Darcian velocity and particle dia. When the Peclet number is sufficiently high, the expression (Eq. 36) reduces to

$$\frac{(k_{dis})_{xx}}{k_f}\simeq 4.5\left(\frac{\varepsilon(1-\varepsilon)^{1/2}}{1.75}\right)Pe_d\simeq 0.8Pe_d \quad (37a)$$

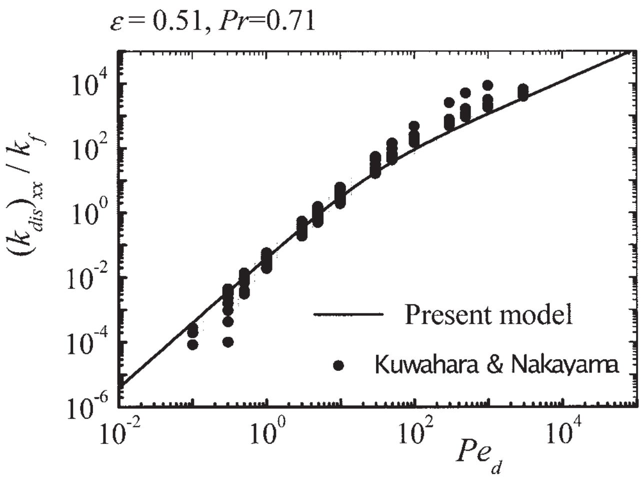

Figure 2. Present model compared with available numerical experiments.

for the packed bed with $\varepsilon \simeq 0.4$. The resulting expression is almost identical to those empirically proposed by Yagi et al.[2] and Vortmeter,[7] namely,

$$\frac{(k_{dis})_{xx}}{k_f} \simeq (0.7 \sim 0.8)\, Pe_d \text{ (Yagi et al.}^2) \tag{38a}$$

$$\frac{(k_{dis})_{xx}}{k_f} \simeq 0.8 Pe_d \text{ (Vortmeter}^7) \tag{38b}$$

Kuwahara and Nakayama[10] carried out an exhaustive numerical experiment to establish the correlations for thermal dispersion coefficients. In Figure 2, their numerical results for the case of $\varepsilon \simeq 0.51$ and $Pr = 0.71$ are compared against our expression (Eq. 36), which, for this case, reduces to

$$\frac{(k_{dis})_{xx}}{k_f} \simeq \frac{0.022 Pe_d^2}{0.71 + 0.024 Pe_d} \tag{39}$$

Figure 2 shows that this expression agrees reasonably well with the numerical experiments for a wide range of Peclet number. The asymptotic behavior, namely, $k_{dis} \propto Pe_d^2$ for the low Peclet number range (as suggested by Taylor[3] and Aris[4]), while $k_{dis} \propto Pe_d$ for the high Peclet number range (as observed in disordered porous media, e.g. [2,7]), has been captured clearly by this expression.

Conclusions

In this study, the dispersion heat flux transport equation for transport in porous media has been derived from the continuity equation, Navier-Stokes equation and energy equation, using the volume averaging theory. An algebraic model for dispersion heat flux has been established by simplifying the resulting transport equation. This algebraic model consistent with the gradient-type diffusion hypothesis agrees very well with the experimental and theoretical evidences.

Notation

A = surface area
A_{int} = interface between the fluid and solid
a_f = specific surface area
b = Forchheimer constant
C_1, C_2, C_{xx}, C_{yy} = turbulence model constants
c_p = specific heat at constant pressure
d = particle dia.
h_f = interfacial heat-transfer coefficient
k = thermal conductivity
K = permeability
p = pressure
$Pe_d = u_D d/\alpha$ = Peclet number based on Darian velocity and particle diameter
$Re_d = u_D d/\nu_f$ = Reynolds number based on Darian velocity and particle diameter
T = temperature
u_i = velocity vector
u, v = velocity components
u_D = Darcian velocity
V, V_c = representative elementary volume
x, y = Cartesian coordinates
a = thermal diffusivity
ε = porosity
ν = kinematic viscosity
ρ = density

Special symbols

$\tilde{\phi}$ = deviation from intrinsic average
$\langle\phi\rangle$ = volume average
$\langle\phi\rangle^{f,s}$ = intrinsic average

Subscripts and superscripts

dis = dispersion
f = fluid
s = solid

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